

Factorization and infrared properties of non-perturbative contributions to DIS structure functions

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In this paper we present a new derivation of the QCD factorization. We deduce the k_T - and collinear factorizations for the DIS structure functions by consecutive reductions of a more general theoretical construction. We begin by studying the amplitude of the forward Compton scattering off a hadron target, representing this amplitude as a set of convolutions of two blobs connected by the simplest, two-parton intermediate states. Each blob in the convolutions can contain both the perturbative and non-perturbative contributions. We formulate conditions for separating the perturbative and non-perturbative contributions and attributing them to the different blobs. After that the convolutions correspond to the QCD factorization. Then we reduce this totally unintegrated (basic) factorization first to the k_T - factorization and finally to the collinear factorization. In order to yield a finite expression for the Compton amplitude, the integration over the loop momentum in the basic factorization must be free of both ultraviolet and infrared singularities. This obvious mathematical requirement leads to theoretical restrictions on the non-perturbative contributions (parton distributions) to the Compton amplitude and the DIS structure functions related to the Compton amplitude through the Optical theorem. In particular, our analysis excludes the use of the singular factors x^{-a} (with $a > 0$) in the fits for the quark and gluon distributions because such factors contradict to the integrability of the basic convolutions for the Compton amplitude. This restriction is valid for all DIS structure functions in the framework of both the k_T - factorization and the collinear factorization if we attribute the perturbative contributions only to the upper blob. The restrictions on the non-perturbative contributions obtained in the present paper can easily be extended to other QCD processes where the factorization is exploited.

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I. INTRODUCTION

Factorization is the basic concept for providing the theoretical grounds for the use of perturbative QCD in the analysis of hadronic reactions. According to it, QCD -calculations can be done in two steps: first, the perturbative effects are accounted for with the calculations of Feynman graphs involved and second, the perturbative results are complemented by the appropriate non-perturbative contributions. There are two basic kinds of the factorization available in the literature: the collinear factorization[1] and k_T -factorization introduced in Ref. [2]. In the framework of the collinear factorization, the border between those stages is established by introducing a factorization scale μ which at the same time acts as the infrared (IR) cut-off to regulate the IR divergences of the perturbative contributions. This kind of factorization is used in DGLAP[3] where the transverse parton momenta $k_{i\perp}$ in the perturbative region are ordered¹ as:

$$\mu^2 < k_{1\perp}^2 < k_{2\perp}^2 < \dots < Q^2. \quad (1)$$

Due to the DGLAP -ordering (1), the virtualities of the initial partons are smaller than the virtualities of the ladder partons. The ordering (1) and the DGLAP evolution equations were introduced in the kinematics where $x = Q^2/2pq \lesssim 1$.

In order to obtain a generalization of the DGLAP equations in the small- x region, the ordering (1) for the virtual quark and gluons in the perturbative region should be replaced by the ordering obtained in Ref. [4] :

$$\mu^2 < \frac{k_{1\perp}^2}{\beta_1} < \frac{k_{2\perp}^2}{\beta_2} < \dots < w = 2pq. \quad (2)$$

We have used in Eqs. (1, 2) the standard notations: p stands for the moment of the initial hadron, q is the moment of the virtual photon, $Q^2 = -q^2$, β_r ($r = 1, 2, \dots$) stand for the fractional longitudinal momenta of the ladder partons and the factorization scale μ^2 is the starting point of the Q^2 -evolution. The ordering (1) is used for accounting for the leading logarithms of Q^2 to all orders in α_s while (2) is used for the total resummation of all leading logarithms regardless of their arguments. The difference between the orderings (1) and (2) leads also to different treatments of α_s (see Refs. [15]-[19] for detail).

The common feature of Eqs. (1) and (2) is that the transverse momenta $k_{i\perp}$ are restricted from below. This is necessary in order to regulate the infrared (IR) divergences arising from the double-logarithmic (DL) contributions. In this case μ acts also as an IR cut-off. Evolving the scattering amplitudes with respect to μ is the essence of the Infrared Evolution Equations (IREE) obtained first in Ref. [5] for the quark scattering and then generalized to various other high-energy processes (see e.g. Ref. [6]). In particular, a generalization of DGLAP was obtained in order to describe the DIS structure functions g_1 and F_1^{NS} at arbitrary x and Q^2 (see the overview of these results in Ref. [7]).

DL contributions are absent in the BFKL equations[8], so this approach is IR-stable. As a consequence, the transverse momenta of the virtual partons here can be arbitrary small. On the other hand, the initial and final partons (gluons) in this approach are essentially off-shell. So, instead of μ , the transverse momenta k_\perp of the external gluons act in BFKL as a new factorization scale, i.e. the border separating the perturbative part from the non-perturbative one. Such a factorization (the k_T -factorization) was suggested in Ref. [2]. The value of k_\perp is arbitrary, so the k_T -factorization involves the integration over k_\perp .

The value of the IR cut-off μ is arbitrary (except for the requirement $\Lambda_{QCD}^2 < \mu^2 \ll Q^2$) but the final expressions for the scattering amplitudes and structure functions should be insensitive to μ , though both perturbative and non-perturbative contributions taken alone depend on it. The sensitivity of various physical observables to IR cut-offs has been a subject of a great interest among theorists. However, such investigations were often focused on the perturbative parts of the observables, leaving the non-perturbative parts unconsidered (see e.g. the recent overview [9] and Refs. therein). In contrast, we consider in the present paper the IR- properties of both perturbative and non-perturbative parts.

As it is well-known, the factorization means that any scattering amplitude and any DIS structure function can be represented as a convolution of the perturbative and non-perturbative blobs. In the present paper we consider the photon-hadron (Compton) scattering and begin with analysis of the forward (at $t = 0$) Compton amplitude $A_{\mu\nu}$. Representing $A_{\mu\nu}$ as a convolution of two blobs depicted in Fig. 1, we study its integrability and then, using the Optical theorem, we apply the obtained results to the DIS structure functions. In contrast, most of the preceding approaches (see e.g. Ref. [10]) addressed directly to the DIS hadronic tensor $W_{\mu\nu}$ and because of that they could not obtain our results and arrive at the conclusions we make in the present paper. The intermediate particles in Fig. 1

¹ The numeration of $k_{i\perp}$ in (1, 2) runs from the bottom to the top of the perturbative ladders.

can be quarks or gluons. The number of them can be arbitrary. We consider only two-particle states connecting the blobs. On one hand, it is the simplest option; on the other hand, it corresponds (at large Q^2) to the leading twist approximation. All blobs in Fig. 1 are not cut. The upper blob in Fig. 1 is supposedly obtained with any perturbative approach, including DGLAP or BFKL, or IREE evolution equations, though strictly speaking it can contain also unperturbative contributions. We study this issue in detail in Sects. II,III.

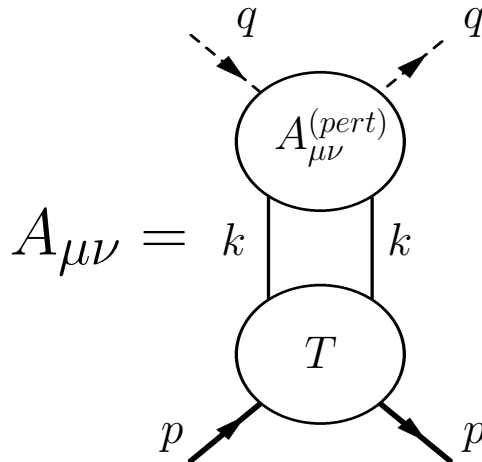


FIG. 1. Representation of $A_{\mu\nu}$ through the convolution of the perturbative and non-perturbative blobs.

The lower blob T is also a mixture of the perturbative and non-perturbative contributions. When the radiative corrections to the upper blob are neglected and the blob is considered in the Born approximation (see Fig. 2), the lower blob can be regarded as altogether non-perturbative. For example, this takes place in the parton model. With the QCD radiative corrections taken into account, both the upper and lower blobs in Fig. 1 acquire the perturbative contributions. Nevertheless, following the standard terminology, throughout the present paper we address the lower (upper) blob as the non-perturbative (perturbative) blob. When the graph in Fig. 1 is cut in the s -channel, i.e. when all s -channel intermediate particles are on-shell, the lower blob $\equiv \Im_s T$ corresponds to the probability to find a constituent (a quark or a gluon) in the incoming hadron. Throughout the paper we will address T and $\Im T$ as the unintegrated parton distributions. When the graph is not cut, we define T as the result of applying the Dispersion Relations to $\Im T$. In the present paper we use the Feynman gauge for virtual gluons. On the other hand, we would like to stress that we consider the inclusive processes only, which makes us free of the analysis of the rapidity divergencies arising when the semi-inclusive processes are investigated. This kind of divergences was first investigated in Refs. [11, 12]. See also recent papers [13] and references therein.

The available in the literature expressions for the unintegrated parton distributions were obtained without using any theoretical grounds. The only argumentation for fixing them was to fit the experimental data. In the present paper we obtain certain theoretical restrictions on the distributions following from the integrability of the convolution in Fig. 1, i.e. from the obvious mathematical requirement that the integration over momentum k must be convergent. In particular, we show that such restrictions are compatible with the use of the singular factors $\sim x^{-a}$ in the fits for the initial parton densities for the singlet structure function F_1 , providing $a < 1$, and exclude such factors in the initial parton densities for the other structure functions.

Our paper is organized as follows: In Sect. II we consider the convolution shown in Fig. 1. It represents the forward Compton amplitude $A_{\mu\nu}$ in a factorized form. We express this amplitude in terms of the invariant amplitudes and, using the Optical theorem, we relate them in the standard way to the DIS structure functions. In order to make our reasoning simpler, we discuss in Sect. III the integrability of the convolution in Fig. 1 when the Born approximation is used for the upper blob. In Sect. IV we consider the convolution in Fig. 1 beyond the Born approximation. In this Sect. we reduce this convolution to the convolution corresponding to the QCD factorization where the upper blob is free of non-perturbative contributions. This factorization involves convolutions with totally unintegrated parton distributions, so we name it the basic form of factorization and discuss the similarity and difference between it and the standard (k_T and collinear) factorizations. In Sect. V we study the impact of the radiative corrections on the integrability requirements obtained in Sect. III. In Sect. VI we show that the basic factorization can be reduced to the k_T -factorization only approximately and formulate the condition for such a reduction. In Sect. VII we continue studying the k_T -factorization and consider restrictions on the parton distributions. In Sect. VIII we reduce the k_T -factorization to the collinear factorization and formulate restrictions on the singular factors $\sim x^{-a}$ in the DGLAP fits

for the initial parton densities for the DIS structure functions following from the integrability of the basic convolutions. Finally, Sect. IX is for concluding remarks.

II. REPRESENTATION OF THE FORWARD COMPTON AMPLITUDE AS A CONVOLUTION

We start by considering the forward Compton scattering off a hadron. The amplitude $A_{\mu\nu}$ for this process includes both the perturbative and non-perturbative contributions. According to factorization, $A_{\mu\nu}$ can be represented as the sum of the convolution shown in Fig. 1 where the intermediate partons are either quarks or gluons:

$$A_{\mu\nu} = A_{\mu\nu}^{(q)} + A_{\mu\nu}^{(g)}, \quad (3)$$

with

$$\begin{aligned} A_{\mu\nu}^{(q)}(p, q, S_h) &= \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 + i\epsilon)^2} \hat{k} \tilde{A}_{\mu\nu}^{(q)}(q, k) \hat{k} T_{\rho\sigma}^{(q)}(k, p, m_h, S_h) \\ A_{\mu\nu}^{(g)}(p, q, S_h) &= \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 + i\epsilon)^2} \tilde{A}_{\mu\nu\rho\sigma}^{(g)}(q, k) T_{\rho\sigma}^{(g)}(k, p, m_h, S_h) \end{aligned} \quad (4)$$

where we have used the superscript q (g) to mark that the intermediate partons in Fig. 1 are quarks (gluons). In Eq. (4) $\tilde{A}_{\mu\nu}^{(q)}$ and $\tilde{A}_{\mu\nu\rho\sigma}^{(g)}$ stand for the upper blobs in Fig. 1 and $T_{\rho\sigma}^{(q)}$, $T_{\rho\sigma}^{(g)}$ denote the lower blobs. Following the conventional notations, we have used the subscripts μ and ν for the polarizations of the external virtual photon; ρ, σ mark polarizations of the intermediate gluons; q, p and k are momenta of the photon, hadron and intermediate virtual partons (quarks or gluons) respectively, m_h and S_h are the hadron mass and spin. In what follows we will focus on the integrations over k , so we skip m_h and S_h in all subsequent formulae. Throughout the paper we will keep the standard notations $Q^2 = -q^2$, $w = 2pq$ and $x = Q^2/w$.

Obviously, even at large Q^2 and w the amplitudes $\tilde{A}_{\mu\nu}^{(q)}$, $\tilde{A}_{\mu\nu\rho\sigma}^{(g)}$ can acquire unperturbative contributions at small k^2 . In order to keep these amplitudes completely perturbative, such a soft region should be excluded from Eq. (4). For example, in the framework of the collinear factorization it is done by introducing the factorization scale. In contrast, we will do it with imposing restrictions on the k^2 -dependence of $T_{\rho\sigma}^{(q)}$, $T_{\rho\sigma}^{(g)}$ but even before doing so we will address $\tilde{A}_{\mu\nu}^{(q)}$, $\tilde{A}_{\mu\nu\rho\sigma}^{(g)}$ as perturbative objects. Besides them, the convolution in Eq. (4) involves the amplitudes $T_{\rho\sigma}^{(q)}$ and $T_{\rho\sigma}^{(g)}$ corresponding to the lower blob in Fig. 1 where non-perturbative contributions are collected. Strictly speaking, $T_{\rho\sigma}^{(q,g)}$ can include perturbative contributions as well. However, these perturbative terms are quite similar to the perturbative contents of the amplitudes $\tilde{A}_{\mu\nu}^{(q)}$, $\tilde{A}_{\mu\nu\rho\sigma}^{(g)}$. Throughout the paper we focus on the non-perturbative contents of $T_{\rho\sigma}^{(q)}$, $T_{\rho\sigma}^{(g)}$, so we address them as a non-perturbative objects. As it is known, the perturbative amplitudes $\tilde{A}_{\mu\nu}^{(q)}(q, k)$, $\tilde{A}_{\mu\nu\rho\sigma}^{(g)}(q, k)$ can contain IR-divergent contributions. The virtual parton momentum k regulates these divergences and k^2 (or k_\perp^2) acts as an IR cut-off. We stress that the convolutions in Eq. (4), though correspond to a factorization of $A_{\mu\nu}$ into the blobs, do not coincide with either the collinear or k_T -factorizations. Moreover, they do not imply any separation of the perturbative contributions from the non-perturbative ones, which is the fundamental concept of the QCD factorization. Because of that we will not apply the term "factorization" to the convolutions in Eq. (4) before obtaining conditions to separate the perturbative and non-perturbative contributions. Instead, we call them the primary convolutions.

The imaginary part (with respect to $s = (q + p)^2$) of $A_{\mu\nu}$ is related to the hadronic tensor $W_{\mu\nu}$:

$$W_{\mu\nu} = \frac{1}{\pi} \Im_s A_{\mu\nu}. \quad (5)$$

In order to simplify the tensor structure of $A_{\mu\nu}$, one can use the conventional DIS projection operators $P_{\mu\nu}^{(r)}$ enlisted in Appendix B. Using them, we represent $A_{\mu\nu}$ in terms of invariant amplitudes A_r :

$$A_{\mu\nu} = \sum_r P_{\mu\nu}^{(r)} A_r. \quad (6)$$

Therefore every DIS structure function f_r is related to the invariant Compton amplitude A_r :

$$f_r = \frac{1}{\pi} \Im A_r. \quad (7)$$

Obviously, each invariant amplitude A_r can be expressed through the primary convolutions of the perturbative invariant amplitudes $\tilde{A}_r^{(q,g)}$ and the non-perturbative invariant amplitudes $T^{q,g}$. Borrowing the terminology of the

Regge theory, we can state that there are the singlet (A_S) and non-singlet (A_{NS}) invariant amplitudes contributing to $A_{\mu\nu}$. They have the vacuum and non-vacuum quantum numbers in the t -channel respectively. It is important that they exhibit different behavior with respect to s : A_S have an extra power of s compared to A_{NS} . Generically, we can write the primary convolutions (4) for them as follows:

$$A_S = \tilde{A}_S \otimes T_S, \quad A_{NS} = \tilde{A}_{NS} \otimes T_{NS}. \quad (8)$$

Applying Eq. (7) to the singlet amplitude A_S yields the singlet structure function F_1 whereas the non-singlet amplitudes A_{NS} lead to other structure functions. We would like to stress that this terminology is not quite adequate. Indeed, among these "non-singlets" there are the amplitudes with imaginary parts corresponding to the flavor singlet components of the structure functions (for example, the singlets $g_{1,2}$).

III. COMPTON AMPLITUDE $A_{\mu\nu}$ IN THE BORN APPROXIMATIONS

In this Sect. we consider the primary convolutions (4) in the simplest case when the perturbative amplitudes $\tilde{A}_{\mu\nu}^{(q)}$, $\tilde{A}_{\mu\nu\rho\sigma}^{(g)}$ in Eq. (4) are calculated in the lowest-order approximation. The convolution for $A_{\mu\nu}$ is depicted in Figs. 2,3. It is convenient to consider this approximation for $\tilde{A}_{\mu\nu}^{(q)}$ and $\tilde{A}_{\mu\nu\rho\sigma}^{(g)}$ severally.

A. Lowest-order approximation for $A_{\mu\nu}^{(q)}$

The case when the amplitude $A_{\mu\nu}^{(q)}$ is calculated in the Born approximation is depicted in Fig. 2.

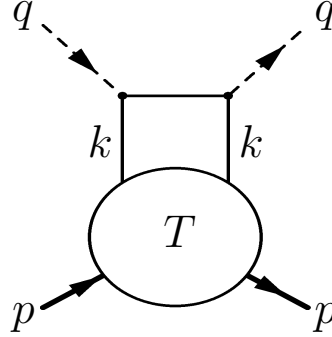


FIG. 2. Born approximation for the amplitude of the forward Compton scattering.

It corresponds to the use of the parton model. Dropping the QED quark-photon coupling e_q^2 as an unessential factor, we can write the Born perturbative contribution $\tilde{A}_{\mu\nu}^{(q\ B)}$ as follows:

$$\tilde{A}_{\mu\nu}^{(q\ B)} = \gamma_\nu \frac{1}{\hat{k} + \hat{q} + i\epsilon} \gamma_\mu + \gamma_\mu \frac{1}{\hat{k} - \hat{q} + i\epsilon} \gamma_\nu \quad (9)$$

where we have neglected the quark mass. Therefore, $\hat{k} \tilde{A}_{\mu\nu}^{(q\ B)} \hat{k}$ can be written as follows:

$$\hat{k} \tilde{A}_{\mu\nu}^{(q\ B)} \hat{k} = \frac{B_{\mu\nu}(q, k)}{(k + q)^2 + i\epsilon} + \frac{B_{\nu\mu}(q, k)}{(k - q)^2 + i\epsilon}. \quad (10)$$

The factors $B_{\mu\nu}(q, k)$ in Eq. (10) are considered in detail in Appendix A. Before performing a detailed calculation, let us give a simple Euclidean-like estimate for the ultraviolet behavior of $T^{(q)}$ in Eq. (4) when $\tilde{A}_{\mu\nu}^{(q)}$ is given by its Born value. Obviously, we can write $d^4k = k^3 dk d\Omega_4$. Then, at large k the Born amplitude in Eq. (9) $\tilde{A}_{\mu\nu}^{(q\ B)} \sim 1/k^3$ and therefore at large k

$$A_{\mu\nu}^{(q)} \sim \int dk \frac{k^3}{k^3} T^{(q)}(k). \quad (11)$$

This integral is convergent at large k only if

$$T^{(q)} \sim k^{-1-h}, \quad (12)$$

with $h > 0$.

Now let us investigate this case more carefully. It is convenient to use the Sudakov variables introduced in Ref. [14]. We will use them in the following form ²:

$$k = -\alpha q' + \beta p + k_\perp, \quad (13)$$

with $q' = q + xp$, $x = Q^2/w$, so that $q'^2 \approx 0$ and

$$k^2 = -w\alpha\beta - k_\perp^2, \quad 2pk = -w\alpha, \quad 2qk = w(\beta + x\alpha), \quad (q + k)^2 = w(\beta - x)(1 - \alpha) - k_\perp^2. \quad (14)$$

$B_{\mu\nu}$ can be simplified with using the projection operators $P_{\mu\nu}^{(r)}$ defined in Appendix B. Then we arrive at the following expressions for $A_{\mu\nu}^B$:

$$A_{\mu\nu}^{(q)} = \sum_r P_{\mu\nu}^{(r)} A_r^B = \sum_r P_{\mu\nu}^{(r)} \int dk_\perp^2 d\beta d\alpha \tilde{A}_r^{(q\ B)}(q, k) \frac{B}{(w\alpha\beta + k_\perp^2 - i\epsilon)^2} T_r^{(q)}(k, p), \quad (15)$$

where $\tilde{A}_r^{(q\ B)}$ are the invariant amplitudes in the Born approximation and B is defined in Eq. (A8). We have also provided the non-perturbative amplitudes T^q with the superscript r as they can be different for different $A_r T^q$. Obviously, $T_r^{(q)}(k, p)$ depend on the invariant energy and the external virtualities:

$$T_r^{(q)}(k, p) = T_r^{(q)}((k + p)^2, k^2) = T_r^{(q)}(w\alpha, (w\alpha\beta + k_\perp^2)). \quad (16)$$

The Born invariant amplitudes $\tilde{A}_r^{(q\ B)}$ are well-known. For example, the amplitudes $\tilde{A}_1^{(q\ B)}$ and $\tilde{A}_3^{(q\ B)}$ related to the structure functions F_1 and g_1 respectively are

$$\begin{aligned} \tilde{A}_1^{(q\ B)} &= \left[\frac{w(1 - \alpha)}{w\beta - Q^2 - w\alpha\beta - k_\perp^2 + i\epsilon} + \frac{-w(1 - \alpha)}{-w\beta - Q^2 - w\alpha\beta - k_\perp^2 + i\epsilon} \right], \\ \tilde{A}_3^{(q\ B)} &= \left[\frac{w(1 - \alpha)}{w\beta - Q^2 - w\alpha\beta - k_\perp^2 + i\epsilon} - \frac{-w(1 - \alpha)}{-w\beta - Q^2 - w\alpha\beta - k_\perp^2 + i\epsilon} \right]. \end{aligned} \quad (17)$$

Taking the imaginary part of $\tilde{A}_1^{(q\ B)}$, $\tilde{A}_3^{(q\ B)}$ with respect to w and neglecting the virtuality $k^2 = -w\alpha\beta - k_\perp^2$, we arrive at the well-known result of the parton model: $\Im \tilde{A}_{1,3}^{(q\ B)} \sim \delta(\beta - x)$.

Now let us integrate Eq. (15) with respect to α and focus on the region of large $|\alpha|$. In this region the amplitudes $\tilde{A}_r^{(q\ B)} \sim \alpha/\alpha$, both the denominator and B in Eq. (15) are $\sim \alpha^2$, so the α -integration looks at large $|\alpha|$ as follows:

$$\int d\alpha \frac{\alpha^3}{\alpha^3} T_r^{(q)}(\alpha) \quad (18)$$

This integral is convergent at large $|\alpha|$ only if

$$T_r^{(q)} \sim \alpha^{-1-h}, \quad (19)$$

with $h > 0$. Eq. (19) confirms the estimate made in Eq. (12). According to Eq. (13), the invariant energy $s' = (p - k)^2$ of $T^{(r)}$ at large α is $s' = w\alpha - w\alpha\beta - k_\perp^2 \approx w\alpha$. So the meaning of Eq. (19) is that $T_r^{(q)}(s')$ should decrease faster than $1/s'$ with growth of s' in order to prevent UV divergency in Eq. (15). Obviously, $\tilde{A}_1^{(q\ B)}$ has non-vacuum quantum numbers in the t -channel and therefore it contributes to the non-singlet part of the Compton amplitude $A_{\mu\nu}$.

² We account for Q^2 but neglect the quark mass.

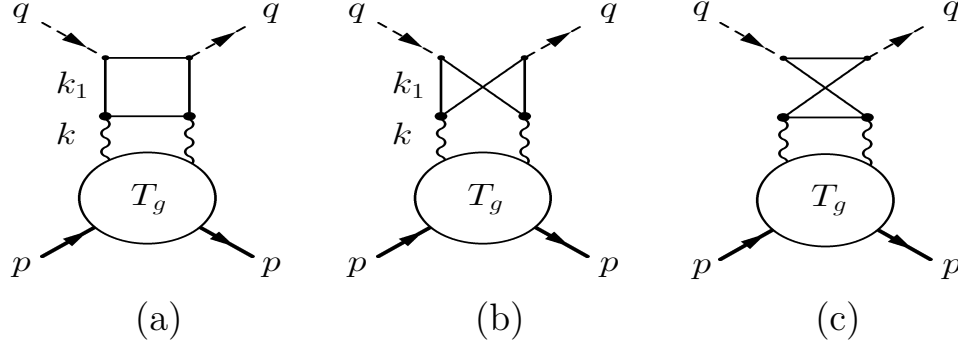


FIG. 3. Gluon contribution to the forward Compton scattering in the lowest order.

B. Lowest-order approximation for $A_{\mu\nu}^{(g)}$

The case when the amplitude $A_{\mu\nu}^{(g)}$ is calculated in the lowest order is depicted in Fig. 3.

For the shortness reason, we will address it as the Born approximation as well. The primary convolution for the Compton amplitude $A_{\mu\nu}^{(g)}$ in this case is

$$A_{\mu\nu}^{(g)}(q, k) = \int \frac{d^4 k}{(2\pi)^4} \Gamma_{\mu\nu \rho\sigma}(q, k) \frac{1}{[k^2 + i\epsilon]^2} T_{\rho\sigma}^{(g)}(k, p) \quad (20)$$

where we have used the notation $\Gamma_{\mu\nu \rho\sigma}(q, k)$ for the upper, perturbative blob given by the quark boxes, one of them depicted in Fig. 3. After applying the projection operators, Eq. (20) can generically be rewritten in terms of the invariant amplitudes as follows:

$$A^{(g)}(q, p) = \int \frac{d^4 k}{(2\pi)^4} \Gamma_{\rho\sigma}(q, k) \frac{1}{[k^2 + i\epsilon]^2} T_{\rho\sigma}^{(g)}(k, p). \quad (21)$$

The rough estimate for $T_{\rho\sigma}^{(g)}$ can be obtained similarly to the one in Eq. (12). Writing $d^4 k = k^3 dk d\Omega_4$ and assuming that k is large leads to

$$A^{(g)} \sim \int \frac{dk}{k} T_{\rho\sigma}^{(g)}(k, p). \quad (22)$$

The integration over k is free of the UV singularity when

$$T_{\rho\sigma}^{(g)} \sim k^{-h}, \quad (23)$$

which differs from Eq. (12).

Now let us make a more detailed estimate of the same Born case, using the Sudakov parametrization (13) for k . At large α Eq. (21) is

$$A^{(g)} \sim \int d\alpha \Gamma(q, k)_{\rho\sigma} \frac{1}{\alpha^2} T_{\rho\sigma}^{(g)}(\alpha, k^2). \quad (24)$$

The contribution α^2 in Eq. (24) comes from k^2 . According to Eq. (13), $k^2 = -w\alpha\beta - k_{1\perp}^2$, i.e. $k_1^2 \sim \alpha$ at large α . The perturbative factor $\Gamma_{\rho\sigma}$ is known to be free of the UV singularities and can be calculated by integrating over momentum k_1 (see Fig. 3). This integration yields different results for the singlet and non-singlet. Also a dependence of $T_{\rho\sigma}^{(g)}$ on α can be different for the singlet and non-singlet.

In the singlet case the polarizations ρ and σ are longitudinal and $T_{\rho\sigma}^{(g)}$ can be represented as follows:

$$T_{\rho\sigma}^{(g)} = \frac{2p_\rho p_\sigma}{w} T_S^{(g)}(\alpha, k^2), \quad (25)$$

so the integration over α in Eq. (21) is reduced to

$$A_S^{(g)} \sim \int d\alpha \frac{\alpha}{\alpha^2} T_S^{(g)}(\alpha, k^2). \quad (26)$$

This integral is convergent when

$$T_S^{(g)} \sim \alpha^{-h}. \quad (27)$$

It coincides with the rough integrability requirement in Eq. (23) but differs from Eq. (12) for the quark contribution.

The non-singlet amplitudes A_{NS} are either flavor non-singlets or spin-dependent. In the latter case $T_{\rho\sigma}^{(g)}$ is anti-symmetrical in $\rho\sigma$ and can be represented as

$$T_{\rho\sigma}^{(g)} = i \frac{m_h}{w} \epsilon_{\rho\sigma\lambda\tau} S_\lambda k_\tau T_{NS}^{(g)}(\alpha, k^2), \quad (28)$$

with S_λ, m_h being the spin of the hadron target and the mass scale respectively, or in a similar form. Therefore, in the non-singlet case Eq. (27) is replaced by

$$\int d\alpha \frac{\alpha^2}{\alpha^2} T_{NS}^{(g)}(\alpha, k^2). \quad (29)$$

This integral is convergent when

$$T_{NS}^{(g)} \sim \alpha^{-1-h}, \quad (30)$$

which coincides with the integrability requirement for the quark non-singlet case. Therefore, the integrability requirements (19,30) for the non-singlet part of $A_{\mu\nu}$ in the lowest-order approximation do not depend on the kind of the intermediate partons.

IV. COMPTON AMPLITUDES A_S, A_{NS} BEYOND THE BORN APPROXIMATION

When the perturbative amplitudes $\tilde{A}_S, \tilde{A}_{NS}$ in Eq. (8) are calculated in the Born approximation, the amplitudes T_S, T_{NS} can be regarded as completely non-perturbative objects. Studying A_S, A_{NS} beyond the Born approximation brings the radiative corrections to $\tilde{A}_S, \tilde{A}_{NS}$. In principle, the radiative corrections can also be placed into T_S, T_{NS} . It converts the Born invariant amplitudes $\tilde{A}_{1,3}^{(q,B)}$ of Eq. (17) and $\Gamma_{\rho\sigma}$ of Eq. (20) into more involved invariant amplitudes which we generically denote $A_{S,NS}^{(q,g)}$. This can have an impact on the α -dependence of the non-perturbative blobs $T_{S,NS}^{q,g}$ and violate the integrability requirements for the singlet (27) and non-singlet (19,30) amplitudes. In the previous Sect. we showed that the integrability requirements do not depend on the kind of the intermediate partons in the convolutions, so in the present Sect. we focus on the quark amplitudes $A_S^{(q)}$ and will skip the superscripts q, g in the sequential formulae. Each of $A_{S,NS}$ can depend on α through k^2 only. Besides, they include the infrared-sensitive radiative corrections becoming singular at small k^2 . In particular, there are the logarithmic terms $\sim \ln^n(w\beta/k^2), \ln(Q^2/k^2)$. In addition to them, the first-loop radiative corrections to the singlet unpolarized amplitudes A_S yield the infrared-sensitive power term $w\beta/k^2$. As it is known, such a term is originated by the 2-gluon intermediate state, with the gluons having the longitudinal polarizations. Such amplitudes can be written as follows:

$$\tilde{A}_S = (w\beta/k^2) M_S(\ln(w\beta/k^2), \ln(Q^2/k^2)). \quad (31)$$

As written explicitly, M_S in Eq. (31) includes the infrared-sensitive logarithms. Of course, it can also include other, infrared-insensitive corrections. The non-singlet amplitudes $A_{NS}^{q,g}$ do not have the power term $(w\beta/k^2)$, so we write them in the following way:

$$\tilde{A}_{NS} = M_{NS}(\ln(w\beta/k^2), \ln(Q^2/k^2)). \quad (32)$$

In these terms we can rewrite the primary convolutions in Eq. (8) as follows:

$$A_S = \int dk_{\perp}^2 \frac{d\beta}{\beta} d\alpha \left(\frac{w\beta}{k^2} \right) M_S(\ln(w\beta/k^2), \ln(Q^2/k^2)) \frac{B}{(w\alpha\beta + k_{\perp}^2)^2} T_S(w\alpha, k^2) \quad (33)$$

and

$$A_{NS} = \int dk_{\perp}^2 \frac{d\beta}{\beta} d\alpha M_{NS}(\ln(w\beta/k^2), \ln(Q^2/k^2)) \frac{B}{(w\alpha\beta + k_{\perp}^2)^2} T_{NS}(w\alpha, k^2). \quad (34)$$

We remind that each of the amplitudes $M_{S,NS}$ and $T_{S,NS}$ in the convolution (33,34) can contain both perturbative and non-perturbative contributions, so these convolutions (we have addressed them as the primary convolutions) cannot be identified with the factorization convolutions.

A. Converting primary convolutions into factorization convolutions

M_S , M_{NS} in Eqs. (33,34) can be entirely perturbative objects only if the region of small k^2 is excluded from these convolutions. Besides, the integrals in (33,34) should be IR- stable. These goals can be achieved when

$$T_{NS} \sim (k^2)^{\eta} \quad (35)$$

and

$$T_S \sim (k^2)^{1+\eta}, \quad (36)$$

with $\eta > 0$, at small k^2 .

Imposing conditions (35,36) makes the amplitudes $M_{S,NS}$ (i.e. the upper blob in Fig. 1) free of non-perturbative contributions and it also solves the problem of the mass singularities[21]. However the lower blobs, $T_{S,NS}$ can include both the perturbative and non-perturbative contributions. When the perturbative blobs were considered in the Born approximation, the lower blobs were regarded as an absolutely non-perturbative objects. In this paper we advocate the point of view that beyond the Born approximation the lower blobs contain non-perturbative contributions only. Indeed, let us add a radiative correction (for instance, a ladder gluon propagator as shown in Fig. 4a) to the Born graph.

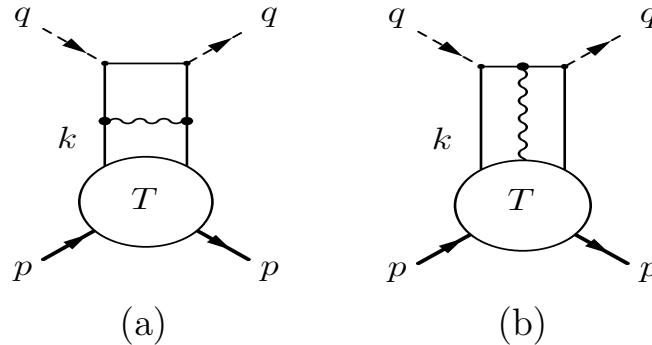


FIG. 4. Radiative corrections to the Born amplitude.

Obviously, it can be included into the upper blob. This procedure can be repeated each time when a new radiative correction is added to Figs. 2,3, except for the case when the new propagators connect the upper and lower blobs as shown in Fig. 4b. Such graphs involve the t -channel intermediate states with three or more partons. Analysis of them requires a special attention (see e.g. Ref. [23]). We do not consider them in the present paper.

Eqs. (33,34) complemented by the IR-restrictions (35, 36) rigorously correspond to the QCD factorization concept where the non-perturbative contributions should be attributed to T_s, T_{NS} while the amplitudes M_S, M_{NS} can be regarded as entirely perturbative objects. Nevertheless, it is clear that the form of the convolutions in these equations does not correspond to the form of either the k_T - or the collinear factorization. Indeed, these factorizations involve the same objects (the perturbative and non-perturbative blobs) as in Eqs. (33,34), however with the smaller number of the integrations. Let us remind that the expressions with the k_T -factorization include two integrations only: over β and k_\perp , while the collinear factorization involves the integration over the longitudinal variable β only. In contrast, Eqs. (33,34) additionally include the integration over α . In order to distinguish the convolutions in Eqs. (33,34) from the convolutions with the k_T - and collinear factorizations, we will call the totally unintegrated form of the factorization convolutions in Eqs. (33,34) the basic form of factorization. Below we show how to proceed from the basic form of factorization to the k_T - and collinear factorizations but before it, let us write down the factorization expressions for the DIS structure functions.

B. Basic form of factorization for the structure functions

Applying Eq. (7) to the invariant Compton amplitudes (34) complemented by the IR-restrictions (35, 36), we arrive at the following expressions for the non-singlet structure functions in the basic form of factorization:

$$f_{NS} = \int dk_\perp^2 \frac{d\beta}{\beta} d\alpha f_{NS}^{(pert)}(\ln(w\beta/k^2), \ln(Q^2/k^2)) \frac{B}{(w\alpha\beta + k_\perp^2)^2} \Omega_{NS}(w\alpha, k^2) \quad (37)$$

where $f_{NS}^{(part)} = (1/\pi)\Im M_{NS}$ stands for the perturbative contributions to the non-singlets and $\Omega_{NS}(w\alpha, k^2) \equiv \Im T_{NS}(w\alpha, k^2)$ are the totally unintegrated non-singlet parton distributions. Similarly, we obtain the following representation for the singlet structure function from (33):

$$f_S = \int dk_\perp^2 \frac{d\beta}{\beta} d\alpha f_S^{(pert)}(\ln(w\beta/k^2), \ln(Q^2/k^2)) \frac{B}{(w\alpha\beta + k_\perp^2)^2} \Omega_s(w\alpha, k^2), \quad (38)$$

where the perturbative singlet contribution is

$$f_S^{(pert)} = \left(\frac{w\beta}{k^2}\right) (1/\pi)\Im M_S. \quad (39)$$

and $\Omega_s(w\alpha, k^2) \equiv \Im T_S(w\alpha, k^2)$ stands for the totally unintegrated singlet parton distributions.

V. CONDITIONS OF INTEGRABILITY FOR NON-SINGLET COMPTON AMPLITUDES

Let us consider the integrations in Eq. (34) for the non-singlet Compton amplitude. Generally, the order of the integrations in Eqs. (33,34) can be arbitrary. It is convenient to integrate over α first. This is usually performed by using the Cauchy theorem. So, the integration line $-\infty < \alpha < \infty$ is complemented by a semi-circle in the upper or lower half of the complex α -plane and after that singularities of the integrand within the α -plane (poles and cuts) should be found. The integral over the semi-circle C_R vanishes when the radius of the semi-circle tends to infinity, so the result is determined by either residues at the poles or integrals along the cuts of the integrand in the α -plane. Now let us consider the integral over α in Eq. (34). For integration along C_R , where $|\alpha| \rightarrow \infty$, the α^2 -terms both in $B(k)$ and in the denominator of Eq. (46) cannot be neglected, so the integral along C_R behaves as

$$\int d\alpha \frac{\alpha^3}{\alpha^3} T_{NS}(\alpha, k_\perp^2). \quad (40)$$

It vanishes at $|\alpha| \rightarrow \infty$ if

$$T_{NS} \sim \alpha^{-1-h} \quad (41)$$

at large α , with arbitrary $h > 0$. This restriction coincides with Eq. (30) obtained in the Born approximation. Therefore, accounting for the radiative corrections to the non-singlet amplitudes does not change the α -dependence of T at large $|\alpha|$ and has the same meaning: T_{NS} should decrease faster than $1/s'$ when the invariant energy $s' = (p-k)^2$ grows.

A. Integrability of the singlet Compton amplitude

The singlet invariant amplitude A_S is defined in Eq. (33). The main difference between Eqs. (33) and the non-singlet amplitudes of Eqs. (34) is the factor $w\beta/k^2$ in Eq. (33). Therefore, the integration over α in Eq. (33) leads to new requirements for the singlet amplitude T_S . Using the same arguments as for T_{NS} , we conclude that the integration over α at $|\alpha| \rightarrow \infty$ looks now as follows:

$$\int d\alpha \frac{\alpha^2}{\alpha^3} T_S(\alpha, k^2). \quad (42)$$

This integral is convergent when

$$T_S \sim \alpha^{-h}, \quad (43)$$

which coincides with the integrability requirement (27) for the singlet amplitude in the lowest-order approximation. It follows from Eq. (13), that $w\alpha = 2pk$ is at large α the invariant energy of the non-perturbative blobs T_S, NS , so Eq. (43), similarly to Eq. (41), predicts the dependence of T_S on the invariant energy $s' = w\alpha$ at large s' : T_S decreases with s' faster than s'^{-h} .

VI. REDUCING THE BASIC FACTORIZATION TO k_T -FACTORIZATION

In order to bring the convolutions in Eqs. (33,34) to the form of the k_T factorization, the integration over α should be performed. Obviously, this integration cannot be performed in the straightforward way without dealing with the perturbative parts of the convolutions because they depend on k^2 and k^2 depends on α through Eq. (13). The only way to bring these convolutions to the form corresponding to the k_T -factorization is to impose the following restriction on the longitudinal component $w\alpha\beta$ of k^2 compared to its transverse component:

$$w\alpha\beta \ll k_\perp^2. \quad (44)$$

Eq. (44) allows us to disentangle the α and β -dependence in Eqs. (33,34,37,38). Under this condition, the upper (perturbative) blob in Fig. 1 approximately depends on β and k_\perp whereas the lower blob approximately depends on α and k_\perp . It allows us to perform the integration over α in Eqs. (33,34,37,38) without involving the perturbative components, which makes possible to bring these equations to the form corresponding to the k_T -factorization. To our knowledge, Eq. (44) is not written explicitly in the literature. However, we would like to stress that it has commonly been used, though in an inexplicit way and in other terms³, in order to obtain integral representations of the DIS structure functions, including DGLAP. At $x \sim 1$, the strong inequality in Eq. (44) should be replaced by the weaker condition $w\alpha\beta \lesssim k_\perp^2$. Eq. (44) perfectly agrees with the well-known fact (but does not follow from it) that the virtualities of the ladder partons are transverse. Applying Eq. (44) to A_{NS} allows us to neglect the α -dependence in the infrared-sensitive terms in M_{NS} and to keep it in T_{NS} only. Performing the integration over α in Eq. (34), we arrive at

$$A_{NS}(q, p) = \int_0^w \frac{dk_\perp^2}{k_\perp^2} \int \frac{d\beta}{\beta} M_{NS}(\ln(w\beta/k_\perp^2), \ln(Q^2/k_\perp^2)) \Psi_{NS}(w\beta, k_\perp^2), \quad (45)$$

with

$$\Psi_{NS}(w\beta, k_\perp^2) = k_\perp^2 \int_{-\infty}^{\infty} d\alpha \frac{BT_{NS}(w\alpha, w\alpha\beta + k_\perp^2)}{(w\alpha\beta + k_\perp^2)^2} \approx \int_{k_\perp^2/w}^{k_\perp^2/w\beta} d\alpha T_{NS}(w\alpha, k_\perp^2). \quad (46)$$

At $x \sim 1$, the upper limit w of the k_\perp^2 -integration in Eq. (45) can be replaced by Q^2 as it takes place in DGLAP.

³ For example, in the book [22] the Sudakov variables α and k_\perp^2 are replaced by $m^2 = (p-k)^2$ and k^2 respectively.

Taking the imaginary part converts Eq. (45) into the following approximate expression for the non-singlet structure functions f_{NS} :

$$f_{NS}(x, Q^2) = \int_0^w \frac{dk_\perp^2}{k_\perp^2} \int_{\beta_0}^1 \frac{d\beta}{\beta} f_{NS}^{(pert)}(w\beta, Q^2, k_\perp^2) \Im \Psi_{NS}(w\beta, k_\perp^2) , \quad (47)$$

with

$$\beta_0 = x + k_\perp^2/w \approx \max[x, k_\perp^2/w] \quad (48)$$

and

$$\Im \Psi_{NS}(w\beta, k_\perp^2, m) \approx \int_{k_\perp^2/w}^{k_\perp^2/w\beta} d\alpha \Im T_{NS}(w\alpha, k_\perp^2, m) \equiv \Phi_{NS}(w\beta, k_\perp^2) \quad (49)$$

where we have introduced the non-singlet unintegrated parton distributions $\Phi_{NS}(\beta, k_\perp^2)$. They correspond to the initial parton densities in the case of the k_\perp -factorization. Now we can represent the non-singlet structure functions in terms of convolutions of the perturbative (partonic) and non-perturbative parts:

$$f_{NS}(x, Q^2) = \int_0^w \frac{dk_\perp^2}{k_\perp^2} \int_{\beta_0}^1 \frac{d\beta}{\beta} f_{NS}^{(pert)}(w\beta, Q^2, k_\perp^2) \Phi_{NS}(w\beta, k_\perp^2) . \quad (50)$$

The structure of Eqs. (45, 50) is quite similar to the one in Ref. [2] and corresponds to the k_T -factorization. Introducing $x_0 = k_\perp^2/w$ and a mass scale m (for example, it can be the hadron target mass) allows us to write Eq. (50) in the following symmetrical form:

$$f_{NS}(x, Q^2) = \int_0^w \frac{dk_\perp^2}{k_\perp^2} \int_{\beta_0}^1 \frac{d\beta}{\beta} f_{NS}^{(pert)}(x/\beta, Q^2/k_\perp^2) \Phi_{NS}(x_0/\beta, k_\perp^2/m^2) . \quad (51)$$

The attractive feature of Eq. (51) is that it exhibits the remarkable symmetry between the arguments of $f_{NS}^{(part)}$ and Φ_{NS} : $x \leftrightarrow x_0$, $Q^2 \leftrightarrow k_\perp^2$ and $k_\perp^2 \leftrightarrow m^2$. Nevertheless, let us remind that though x and Q^2 are conventionally regarded as independent variables, strictly speaking they are not independent. Indeed, they can be regarded as independent only in the case when Q^2 is kept fixed while w is scanned. In the opposite case when w is fixed and Q^2 is varied, the variables Q^2 and x are not independent at all. So, the parametrization of the structure functions in terms of the really independent variables w and Q^2 would be preferable at least for the theoretical analysis.

Similarly to the non-singlet case, the singlet amplitude A_S can be written as follows:

$$A_S(q, p) = \int_0^w \frac{dk_\perp^2}{k_\perp^2} \int \frac{d\beta}{\beta} \left(\frac{w\beta}{k_\perp^2} \right) M_S(\ln(w\beta/k_\perp^2), \ln(Q^2/k_\perp^2)) \Psi_S(w\beta, k_\perp^2), \quad (52)$$

with

$$\Psi_S(w\beta, k_\perp^2) = (k_\perp^2)^2 \int_{-\infty}^{\infty} d\alpha \frac{BT_S(w\alpha, w\alpha\beta + k_\perp^2)}{(w\alpha\beta + k_\perp^2)^3} \approx \int_{k_\perp^2/w}^{k_\perp^2/w\beta} d\alpha TS(w\alpha, k_\perp^2). \quad (53)$$

The unintegrated parton distribution $\Phi(w\beta, k_\perp^2)$ can be defined similarly to the non-singlet case:

$$\Im \Psi_S(w\beta, k_\perp^2, m) \approx \int_{k_\perp^2/w}^{k_\perp^2/w\beta} d\alpha \Im T_S(w\alpha, k_\perp^2, m) \equiv \Phi_S(w\beta, k_\perp^2). \quad (54)$$

Therefore, the singlet structure function f_S (i.e. the DIS structure function F_1^S) is

$$f_S(x, Q^2) = \int_0^w \frac{dk_\perp^2}{k_\perp^2} \int_{\beta_0}^1 \frac{d\beta}{\beta} f_S^{(pert)}(w\beta, Q^2, k_\perp^2) \Phi_S(w\beta, k_\perp^2) . \quad (55)$$

VII. INFRARED BEHAVIOR OF PARTON DISTRIBUTIONS IN THE k_T -FACTORIZATION

The generalized parton distributions $T_{S,NS}$ (in the expressions the Compton amplitudes) and $\Phi_{S,NS}$ (in the expressions the structure functions) include unperturbative contributions. When the k_T -factorization is used, there is no problem about UV divergences but the IR divergences at small k_\perp must be regulated. Below we study the impact of the integrability conditions on the properties of the parton distributions in the framework of the k_T -factorization.

Let us first consider the k_\perp behavior of the non-singlets. The explicit expressions for M_{NS} can be different, depending on the approach chosen to calculate it but their IR-divergent contributions in M_{NS} are always logarithmic:

$$M_{NS} \sim \ln^n(w\beta/k_\perp^2), \quad \ln^l(Q^2/k_\perp^2), \quad (56)$$

with $n, l = 1, 2, \dots$. So, $k_\perp^2 \ll w\beta, Q^2$ in any perturbative approach. It means that the k_\perp -integration is free of ultraviolet divergences. However, the infrared divergences can appear in Eqs. (45,50). In order to avoid them, we have to put a restriction on T_{NS} at small k_\perp :

$$T_{NS} \sim (k_\perp^2)^\eta \quad (57)$$

where $\eta > 0$. The main difference between Eqs. (52,55) and the non-singlets (45,50) is the factor $w\beta/k^2$ in Eqs. (52,55). It changes the non-singlet restriction (57) for

$$T_S \sim (k_\perp^2)^{1+\eta} \quad (58)$$

at small k_\perp . Besides regulating the IR-divergences, Eqs. (57,58) have another important meaning for the k_T factorization: they allow to neglect the region of small k_\perp in the convolutions, solving therefore the problem of the mass singularities[21].

The initial parton densities Φ contribute to the DIS structure functions when the k_\perp -factorization is used. In this case, k_\perp^2/β is their invariant energy, so Eqs. (41, 43), with α replaced by $k_\perp^2/w\beta$, predict the following dependence of the initial parton distributions on β :

$$\Phi_{NS} \sim \beta^h, \quad (59)$$

$$\Phi_S \sim \beta^{-1+h}. \quad (60)$$

They mean that Φ_{NS} should be regular in β when $\beta \rightarrow 0$ and Φ_S can be singular but with $h < 1$. These restrictions and the ones in Eqs. (57, 58) must be respected when fits for the initial parton distributions Φ_S, Φ_{NS} are composed in the framework of the k_T -factorization.

VIII. COLLINEAR FACTORIZATION FOR THE STRUCTURE FUNCTIONS

In this Sect. we consider the infrared properties of the DIS structure functions when the collinear factorization is introduced. Historically, the collinear factorization[1] was the first one considered in the literature. In general, the basic assumption of the collinear factorization is to introduce the scale μ for the parton virtualities k_i^2 , with $k_i^2 > \mu^2 > \Lambda_{QCD}^2$ as an explicit border between the perturbative (hard) and non-perturbative (soft) domains of QCD. For describing the DIS structure functions, this factorization first was used to construct the DGLAP evolution equations where the ordering (1) was suggested to evolve the structure functions from μ^2 to Q^2 . Later, the collinear factorization combined with the ordering (2) was used to describe some of the structure functions in the small- x region where the total resummation of $\ln^k(1/x)$ becomes essential (see Ref. [7] and Refs therein for detail).

A. Transition from the k_T -factorization to the collinear factorization

Eqs. (45,52) for the forward Compton amplitudes as well as Eqs. (50,55) for the structure functions involve integrations over β and k_\perp , whereas similar expressions in the collinear factorization operate with integrations over β only. So, our aim now is to integrate over k_\perp in Eqs. (45,52,50,55) without dealing with the perturbative contributions. However, both the perturbative and unperturbative factors in Eqs. (45,52,50,55) depend on k_\perp . It makes impossible any straightforward k_\perp -integration of the unperturbative terms. Therefore, a transition from the k_\perp -factorization

to the collinear one can be done only approximately. In a sense, it is similar to the transition from the basic form of the factorization to the k_T -factorization we have considered in Sect. VI.

In order to do it, let us first suppose that $\Phi_{S,NS}(\equiv \Phi)$ in Eqs. (50,55) can be represented as

$$\Phi = \varphi(\beta, k_\perp^2) \Theta(\mu^2 - k_\perp^2), \quad (61)$$

with $\varphi(\beta, k_\perp^2)$ decreasing rapidly at large k_\perp and respecting the restrictions in Eqs. (57,58). This choice of Φ is depicted in Fig. 5.

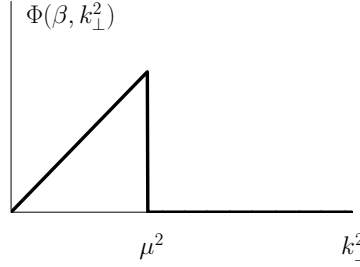


FIG. 5. The plot for $\Phi(\beta, k_\perp^2)$ defined according to Eq. (61)

It means that μ is the strict border between the contributions forming the upper and lower blobs in Fig. 1: the perturbative corrections with $k_\perp > \mu$ are attributed to the upper blob and participate in the orderings of Eqs. (1, 2) while the lower blob includes the contributions with $k_\perp < \mu$.

Alternatively, the transition from the k_T -factorization to the collinear factorization can be done without using the Θ -function. Let us assume (see Fig. 6) that Φ has a sharp maximum at $k_\perp^2 = \mu^2 \ll Q^2$ saturating the integration over k_\perp in Eqs. (50,51).

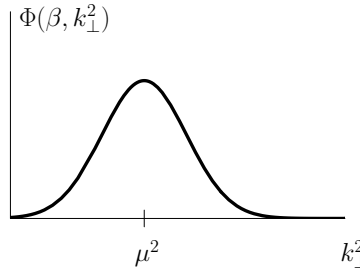


FIG. 6. The plot for $\Phi(\beta, k_\perp^2)$ defined according to Eq. (62)

For example, it takes place when the k_\perp^2 -dependence of Φ includes the Gaussian exponential (or a similar sharply peaked factor). $\Phi(\beta, k_\perp^2)$ can also include a flatter function of k_\perp^2 :

$$\Phi = \varphi(\beta, k_\perp^2) \exp \left[-\sigma (k_\perp^2 - \mu^2)^2 \right] \quad (62)$$

where the function $\varphi(\beta, k_\perp^2)$ in Eq. (62) respects Eqs. (57,58). In this case μ cannot be regarded as the border between the contributions to the upper and lower blobs in Fig. 1: Φ in Eq. (62) comprises the contributions with arbitrary k_\perp .

Substituting Eq. (61) or (62) in Eqs. (45,52,50,55) allows us to integrate Φ over k_\perp^2 independently of $f^{(pert)}$, arriving at the standard expressions with the collinear factorization. In particular, combining (61) or (62) with Eqs. (51, 55) we arise at the following generic expression for the structure functions:

$$f(x, Q^2) = \sum_{r=q,g} \int_{\beta_0(\mu)}^1 \frac{d\beta}{\beta} f_r^{(pert)}(x/\beta, Q^2/\mu^2) \varphi^{(r)}(\beta, \mu^2) \quad (63)$$

where $\beta_0(\mu) = x + \mu^2/w \approx x$. We have used in Eq. (63) the generic notations $f, f_r^{(pert)}$ both for the singlets and non-singlets; the subscripts q, g in Eq. (63) refer to the quark and gluon respectively. The perturbative contributions $f_r^{(pert)}$ in Eq. (63) are usually calculated either in the framework of DGLAP or with total resummation of the leading (and sub-leading) logarithms. The transition from the k_T -factorization to the collinear factorization does not affect the β -dependence in Eqs. (59,60), so the singlet $\varphi_S(\beta, \mu^2)$ and non-singlet $\varphi_{NS}(\beta, \mu^2)$ parton distributions used in the framework of the collinear factorization should obey the following relations respectively:

$$\varphi_{NS}(\beta, \mu^2) \sim \beta^h, \quad (64)$$

$$\varphi_S(\beta, \mu^2) \sim \beta^{-1+h}, \quad (65)$$

with $h > 0$. We would like to stress here that the restrictions (64, 65) are not related to the integrability of the collinear factorization convolution in Eq. (63) in itself. Indeed, the integral in Eq. (63) is convergent at any φ because the integration over β here runs over the compact region $[\beta_0(\mu); 1]$. However, violations of (64, 65) would destroy the integrability of the more general factorization convolutions in Eqs. (33,34). We remind that in the present paper we have derived (64, 65) from Eqs. (33,34).

Let us notice that μ^2 in Eq. (63) stands both for the factorization scale and for the starting point of the perturbative Q^2 -evolution. On the other hand, this evolution can start at an arbitrary point. When it starts at $Q^2 = \mu_0^2$ (with $\mu_0^2 > \mu^2$), Eq. (63) can be written as

$$f(x, Q^2) = \sum_{r=q,g} \int_{\beta_0(\mu)}^1 \frac{d\beta}{\beta} f_r^{(pert)}(x/\beta, Q^2/\mu_0^2) \tilde{\varphi}^{(r)}(\beta, \mu_0^2), \quad (66)$$

with $\tilde{\varphi}^{(r)}(\beta, \mu_0^2)$ being new parton distributions. They are defined at the new factorization scale μ_0^2 . The values of μ_0^2 are located within the range of the Perturbative QCD, so the parton distributions $\tilde{\varphi}^{(r)}(\beta, \mu_0^2)$ include both non-perturbative and perturbative contributions. Although both $f_r^{(pert)}(x/\beta, Q^2/\mu_0^2)$ and $\tilde{\varphi}^{(r)}(\beta, \mu_0^2)$ depend on μ_0^2 , the integral in Eq. (66) is obviously μ_0^2 -independent exactly as it is independent of contributions of any intermediate integration point. Indeed, the inclusion of the perturbative contributions into $\tilde{\varphi}(x, \mu_0^2)$ implies that there is another scale μ (with $\mu < \mu_0$) in $\tilde{\varphi}(x, \mu_0^2)$, so the perturbative methods can be used for the evolution from μ^2 to μ_0^2 . Obviously, the transition from $\varphi_{S,NS}(x, \mu^2)$ to $\tilde{\varphi}_{S,NS}(x, \mu_0^2)$ can be done with the same technique as the evolution from μ_0^2 to Q^2 in $f^{(pert)}$:

$$f(Q^2) = U(Q^2, \mu_0^2) \tilde{\varphi} \otimes (\mu_0^2), \quad \tilde{\varphi}(\mu_0^2) = U(\mu_0^2, \mu^2) \otimes \varphi(\mu^2). \quad (67)$$

where $U(b, a)$ is a generic notation for the evolution operator, with $a(b)$ standing for the starting (final) point of the evolution. We have skipped dependence on all variables in Eq. (67) inessential at the moment. For instance, in the case of the LO DGLAP $\tilde{\varphi}(\mu_0^2)$ and $\varphi(\mu^2)$ are related in the momentum space as follows:

$$\tilde{\varphi}(\omega, \mu_0^2) = \left[\frac{\ln(\mu_0^2/\Lambda^2)}{\ln(\mu^2/\Lambda^2)} \right]^{\gamma^{(1)}(\omega)/b} \varphi(\omega, \mu^2), \quad (68)$$

where $\gamma^{(1)}(\omega)$ is the LO anomalous dimension and $b = [33 - 2n_f]/(12\pi)$ is the first coefficient of the β -function. In this case the perturbative structure functions $f^{(pert)}(\omega, Q^2)$ and $f^{(pert)}(\omega, \mu_0^2)$ are related similarly:

$$f^{(pert)}(\omega, Q^2) = \left[\frac{\ln(Q^2/\Lambda^2)}{\ln(\mu_0^2/\Lambda^2)} \right]^{\gamma^{(1)}(\omega)/b} f^{(pert)}(\omega, \mu_0^2) \quad (69)$$

Substituting Eqs. (68,69) into (66) kills the dependence of $f(x, Q^2)$ on the arbitrary scale μ_0^2 but does not affect its dependence on the scale μ . We remind we have defined the scale μ in Eqs. (61, 62) as the scale separating the perturbative and non-perturbative contributions.

Our restrictions (64,65) look invalid in the framework of the approach suggested in Ref. [23]. We will address it as the EGR-approach. In this approach the factorization scale μ_0 can also be chosen arbitrary and the QCD coupling is kept fixed. The EGR-approach is based on the observation that the ladder Feynman graphs contributing to the Compton amplitude yield the sub-leading IR-stable logarithmic contributions in the kinematic region where the virtualities k_r^2 ($r = 1, 2, \dots$) of the vertical partons are very small:

$$0 < |k_r^2| < \mu_0^2, \quad (70)$$

while virtualities of the horizontal partons are greater than μ_0^2 . Like in Eqs. (1,2), we have numerated the ladder momenta k_r in (70) from the bottom to the top of the ladders graphs. Integrations of the ladder graphs over k_r in the region (70) yield logarithmic contributions $\sim \ln^n x$ for the non-singlets and $\sim (1/x) \ln^n x$ for the singlet. Resummation of them leads to the Regge factors $\sim x^{-a}$ for the non-singlets and $\sim x^{-1-a}$ for the singlet (see Refs. [23, 24] for detail). As k_r^2 in the region (70) are small, these Regge factors, according to Ref. [23] should be included into $\tilde{\varphi}(x, \mu_0^2)$. The sub-leading logarithmic contributions $\sim \ln^n x$ are IR-stable, so the EGR-approach operates with the scale μ_0 only and does not need any additional scale μ . This approach obviously contradicts our approach and makes invalid our restrictions (64,65). We plan to compare our and EGR approaches in detail in our forthcoming paper. Here we just mention that the EGR- approach leads to a much more complicated factorization construction than the one depicted in Fig. 1: instead of two blobs it involves a pile (etagere) of perturbative and non-perturbative blobs.

B. Restrictions on the singular factors in the DGLAP fits

Now let us focus on the convolution in Eq. (63), with the DGLAP expression used for the perturbative structure functions $f_r^{(pert)}$. In the standard notations, Eqs. (63) takes the following form:

$$f(x, Q^2) = \sum_{r=q,g} \int_x^1 \frac{dy}{y} f_{DGLAP}^{(r)}(x/y, Q^2/\mu^2) \delta r(y, \mu^2). \quad (71)$$

where we have dropped the term μ^2/w in the lowest integration limit. The initial parton densities $\delta q(x), \delta g(x)$ in Eq. (71) are obtained by fitting the experimental data at $\mu \sim 1$ GeV. They include both singular factors x^{-a} (with $a > 0$) and regular factors $\sim (1-x)^b, (1+cx^d)$, with $b, c, d > 0$.

Comparing Eqs. (63) and (64,65) with (71) drives us to conclude that the standard DGLAP fits with the singular factors x^{-a} for the non-singlet structure functions are excluded by Eq. (64). On the other hand, Eq. (65) admits the use of the singular factors for the singlet F_1 , however with the exponent $a < 1$. We would like to stress here that the use of the singular factors cannot be excluded by requirements on the integrability of the structure functions (where all integration limits are finite). However, the use of such factors contradicts to the integrability the expressions for the forward Compton amplitudes where the integrations run over the whole phase space. Now let us comment on these results in more detail and consider below both the k_T - and collinear factorizations.

First of all, we infer that the fits for the polarized quark and gluon distributions should satisfy the restriction (59) or (64), depending on whether the k_T - or collinear factorization is used.

Then, the spin-independent non-singlet structure functions involve the unpolarized quark distributions only, so each of the quark distributions should again satisfy Eqs. (59,64). It is true also for the singlet F_2 .

The difference between the restrictions (59,64) for the non-singlets and the ones (60,65) for the singlets arises entirely from the perturbative components of the singlet F_1 ($\equiv F_1^S$) and the other structure functions. F_1^S involves the unpolarized quark and gluon distributions. The quark distributions in F_1^S are the same as the ones to the non-singlets F_1 , so they satisfy Eqs. (59,64) but the restriction on the gluon distribution to F_1^S is softer because it is given by Eqs. (64,65). On the other hand, the same gluon distribution is supposed to contribute to the singlet F_2 and therefore it should also satisfy Eqs. (59, 64). Eventually, we conclude that the singular factors x^{-a} should be absent in all parton distributions.

In Refs. [7, 20] we showed that the singular factors in the DGLAP -fits were introduced to match experimental data at small x . The reason for introducing them is that they cause a steep rise of the structure functions at small x , thereby making possible the use of DGLAP in the small- x region. As a matter of fact, they mimic the total resummation of the leading logarithms of x , which is beyond the reach of DGLAP. With the resummation taken into account, such a rise of the structure functions at small x is achieved automatically, so the singular factors become unnecessary. In addition, we have shown in the present paper that there are theoretical restrictions on the singular factors following from the requirement of integrability of the factorization convolutions for the forward Compton amplitudes.

IX. SUMMARY

In the present paper we have derived the k_T - and collinear factorizations for the Compton amplitude and DIS structure functions and obtained the restrictions on the parton distributions to the DIS structure functions. The parton distributions available in the literature are fixed phenomenologically. In contrast, we have obtained the theoretical restrictions on them, exploiting the obvious mathematical requirement of integrability of the factorization convolutions.

We began by considering the convolutions (4) depicted in Fig 1 where the amplitude of the forward Compton scattering off a hadron target is represented through two blobs connected by the two-parton intermediate states involving either quarks or gluons. Strictly speaking, the convolutions in Eq. (4) do not correspond to the conventional concept of the QCD factorization because each of the amplitudes there (each blob in Fig. 1) can contain the both non-perturbative and perturbative contributions, so we called them the primary convolutions and considered their integrability in the Born approximation. Accounting for the radiative corrections converted the Born amplitudes into Eqs. (33,34). These convolutions complemented by the restrictions in Eqs. (35, 36) rigorously correspond to the QCD factorization concept: the upper blob in Fig. 1 becomes free of non-perturbative contributions while the lower blob is non-perturbative. However, the factorization convolutions in Eqs. (33,34) differ from the convolutions in the k_T - and collinear factorization and involve the totally unintegrated parton distributions, so we call such a factorization the basic factorization. The convolutions in Eqs. (33,34) are UV-stable when the restrictions (41,43) are fulfilled. Applying the Optical theorem to Eqs. (34,33), we arrived at the expressions (37,38) for the singlet and non-singlet structure functions in the basic (totally unintegrated) form.

We showed that reducing the basic factorization to the k_T - factorization can be done only when the approximation in Eq. (44) was accepted. Using this result, we obtained Eqs. (50,55) for the structure functions in the framework of the k_T -factorization. Investigating the impact of the integrability restrictions (41,43) on (50,55), we obtained the theoretical restrictions on the parton distributions at the k_T -factorization: Eqs. (59,60) show that the non-singlet parton distributions should be regular in the longitudinal momentum β while the distributions for the singlet F_1 can include the singular factors β^{-h} , but with $h < 1$. Besides, the parton distributions should be regular in the transverse momenta. These restrictions should be respected in fits for the parton distributions composed in the framework of the k_T -factorization. When the k_\perp -dependence of those parton distributions exhibits a sharp maximum at $k_\perp^2 = \mu^2$, as shown in Figs. 5,6, the k_T -factorization can be reduced to the collinear factorization, with μ playing the role of the factorization scale. This our treatment of μ can be checked by fitting experimental data in the framework of the k_T - factorization.

The integrability of the forward Compton scattering amplitudes at the basic factorization leads to the restrictions (64,65) on the singular factors x^{-a} in the fits for the initial parton distributions: $a < 1$ in the fits for the singlet F_1 and $a \leq 0$ in the fits for other structure functions providing the perturbative corrections are attributed to the upper blob.

There is no essential difference between the restrictions in Eqs. (64,65) and (59,60) obtained for the collinear and k_T - factorizations respectively. On the other hand, the restrictions (60,65) on the unpolarized singlet differ from the restrictions (59,64) for all other parton distributions. However, this difference results altogether from the peculiar ($\sim 1/x$) perturbative x -evolution of F_1 singlet compared to the other structure functions, i.e. from the difference between singlet and non-singlet coefficient functions. So, as the same parton distributions contribute to the singlet F_1 and the other spin-independent structure functions, all the parton distributions should satisfy Eqs. (59,64). The same is true for the polarized parton distributions. The necessity to keep the singular factors in the fits should be regarded as a clear indication that the perturbative components of the structure functions used in the analysis lack the resummation of the contributions $\sim \ln^k(1/x)$.

To conclude, let us notice that the results for the non-perturbative contributions to the Compton amplitude and DIS structure functions obtained in the present paper follow from general properties of the primary convolutions in Eq. (4) and because of that they can easily be applied to other inclusive hadronic processes at high energies.

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Appendix A: Simplification of $B_{\mu\nu}$

The tensor $\hat{k}A_{\mu\nu}^{(q\ B)}\hat{k}$ in Eq. (10) depends on the Lorentz subscripts μ and ν through the term

$$B_{\mu\nu} \equiv \hat{k}\gamma_\nu(\hat{q} + \hat{k})\gamma_\mu\hat{k}. \quad (\text{A1})$$

$B_{\mu\nu}$ contains contributions symmetrical and antisymmetrical with respect μ and ν : $B_{\mu\nu} = B_{\mu\nu}^{(sym)} + B_{\mu\nu}^{(asym)}$. Let us consider first the antisymmetrical part, $B_{\mu\nu}^{(asym)}$. It contributes to the spin-dependent part $\tilde{A}_{\mu\nu}^{(spin)}$ of the amplitude $\tilde{A}_{\mu\nu}^{(pert)}$. $B_{\mu\nu}^{(asym)}$ can be simplified as follows:

$$B_{\mu\nu}^{(asym)} = -i(1 + \alpha)\epsilon_{\mu\nu\lambda\rho}q_\lambda[\gamma_5\hat{k}\gamma_\rho\hat{k}]. \quad (\text{A2})$$

The symmetrical part of $B_{\mu\nu}$ can be simplified similarly. For example, the contribution in $B_{\mu\nu}^{(sym)}$ proportional to $g_{\mu\nu}$ is

$$g_{\mu\nu}(1 + \alpha)q_\rho[\hat{k}\gamma_\rho\hat{k}]. \quad (\text{A3})$$

Now let us simplify $\hat{k}\gamma_\rho\hat{k}$.

Using the Sudakov parametrization $k = \alpha q + \beta p + k_\perp$ in the c.m.f. where $p = \sqrt{w/2}(1, 0, 0, 1)$ and $q = \sqrt{w/2}(1, 0, 0, -1)$, we rewrite $\hat{k}\gamma_\rho\hat{k}$ as follows:

$$\begin{aligned} \hat{k}_\perp\gamma_\rho\hat{k}_\perp &= (\hat{k}_\perp\gamma_\rho\hat{k}_\perp)^L + (\hat{k}_\perp\gamma_\rho\hat{k}_\perp)^T, \\ (\hat{k}_\perp\gamma_\rho\hat{k}_\perp)^L &= (\alpha\hat{q} + \beta\hat{p})\gamma_\rho(\alpha\hat{q} + \beta\hat{p}), \\ (\hat{k}_\perp\gamma_\rho\hat{k}_\perp)^T &= \hat{k}_\perp\gamma_\rho\hat{k}_\perp \end{aligned} \quad (\text{A4})$$

As the subscript ρ runs in the longitudinal space ($\rho = 0, 3$),

$$(\hat{k}\gamma_\rho\hat{k})^T = k_\perp^2\gamma_\rho. \quad (\text{A5})$$

This is the only important term in LLA. When we go beyond the LLA, the longitudinal term can also be essential. In order to deal with it, we represent γ_ρ in terms of \hat{q}, \hat{p} :

$$\gamma_0 = \frac{\hat{p} + \hat{q}}{\sqrt{w}}, \quad \gamma_3 = \frac{\hat{p} - \hat{q}}{\sqrt{w}} \quad (\text{A6})$$

Substituting it into Eq. (A4) and neglecting terms $\sim q^2, p^2$, we obtain

$$\begin{aligned} (\hat{k}_\perp\gamma_\rho\hat{k}_\perp)^L &= \frac{\alpha^2\hat{q}\hat{p}\hat{q} + \beta^2\hat{p}\hat{q}\hat{p}}{\sqrt{w}} = w[(\alpha^2 + \beta^2)\gamma_0 + (-\alpha^2 + \beta^2)\gamma_3], \\ (\hat{k}_\perp\gamma_\rho\hat{k}_\perp)^L &= \frac{\alpha^2\hat{q}\hat{p}\hat{q} - \beta^2\hat{p}\hat{q}\hat{p}}{\sqrt{w}} = w[(\alpha^2 - \beta^2)\gamma_0 + (\alpha^2 + \beta^2)\gamma_3]. \end{aligned} \quad (\text{A7})$$

The contributions $\sim \beta^2$ in Eq. (A7) can be dropped when Leading Logarithmic Approximation is used to calculate $A^{(pert)}$. It follows from Eqs. (A4, A7) that $\hat{k}\gamma_\rho\hat{k} \sim \alpha^2$ when $\alpha \rightarrow \infty$. On the contrary, when α is small, $\hat{k}\gamma_\rho\hat{k} = k_\perp^2\gamma_\rho$. It is convenient to use the interpolation expression for $\hat{k}\gamma_\rho\hat{k}$:

$$\hat{k}\gamma_\rho\hat{k} \approx \gamma_\rho[w(\alpha^2 + \beta^2) + k_\perp^2] \equiv \gamma_\rho B. \quad (\text{A8})$$

It leads to the following approximative expressions for the symmetrical and antisymmetrical parts of $B_{\mu\nu}$:

$$\begin{aligned} B_{\mu\nu}^{(sym)} &= g_{\mu\nu}\hat{q}B, \\ B_{\mu\nu}^{(asym)} &= i\epsilon_{\mu\nu\lambda\rho}q_\lambda\gamma_\rho B \end{aligned} \quad (\text{A9})$$

Appendix B: Projection operators for the Compton amplitude

The standard way of studying $A_{\mu\nu}$ is to introduce the projection operators and expanding $A_{\mu\nu}$ into a series of invariant amplitudes. The spin-independent part $A_{\mu\nu}^{(unpol)}$ of $A_{\mu\nu}$ is parameterized as follows:

$$A_{\mu\nu}^{(unpol)} = P_{\mu\nu}^{(1)} A_1 + P_{\mu\nu}^{(2)} A_2 \quad (B1)$$

where

$$P_{\mu\nu}^{(1)} = \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2} \right), \quad P_{\mu\nu}^{(2)} = \left[P_\mu - q_\mu \frac{pq}{q^2} \right] \left[P_\nu - q_\nu \frac{pq}{q^2} \right] \frac{1}{pq} \quad (B2)$$

while the expansion of the spin-dependent part $A_{\mu\nu}^{(spin)}$ is

$$A_{\mu\nu}^{(spin)} = P_{\mu\nu}^{(3)} A_3 + P_{\mu\nu}^{(4)} A_4, \quad (B3)$$

with

$$P_{\mu\nu}^{(3)} = i\epsilon_{\mu\nu\lambda\rho} q_\lambda S_\rho \frac{m}{pq}, \quad P_{\mu\nu}^{(4)} = i\epsilon_{\mu\nu\lambda\rho} q_\lambda S_\rho \frac{m}{pq} \left[S_\rho - p_\rho \frac{qS}{pq} \right]. \quad (B4)$$

Any of the Compton invariant amplitudes $A_{1,2,3,4}$ includes both the flavor singlet and non-singlet contributions. Their imaginary parts correspond to the singlet and non-singlet components of the DIS structure functions.

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